A model of solid-liquid separation of mining slurries: experimental validation and scale-up

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ABSTRACT

The thickening process is a major concern in the industrial treatment of high solids mining slurries. Despite the longstanding multidisciplinary effort to understand the phenomena involved in the solid-liquid separation, the industrial operation is still largely based on empirical considerations. The main goal of this work is to follow up on a well-established mathematical model of suspensions, providing a new approach focused on industrial applications. To this end, we have developed a computational simulator that uses experimental data from the physical characterization of the pulp, in order to solve numerically the time-dependent behavior of the thickening process.

The current implementation of the code is flexible, allowing for exploration on several aspects including the effect of vessel geometry and varying feed/discharge rates. Common features such as bed height, compression, clarification and concentration profiles can be observed in batch and continuous mode of operation. We have carried out extensive studies, showing in general good agreement of the model vs. experimental data obtained from semi-pilot testing. This analysis has been used to validate the predicting capabilities of the model in an experimental setup.

Subsequently, in association with an industrial plant, we aim to gain on-site insight and validate the integrated model. The extension of this research will contribute to the analysis of different operational conditions and provide a better understanding on possible directions to improve water recovery, quality of disposals and general efficiency of the process.
INTRODUCTION

The thickening process frequently faces numerous difficulties during operation such as low discharge concentration, bad quality overflow and high torque. These conditions can lead to non-compliance of design parameters and insufficient water recovery, causing a significant impact on industrial production. Multidisciplinary research aimed at addressing some these issues has advanced greatly in the past decades; however, thickener control still follows mostly empirical rules and unclear methodologies.

At CI-JRI we intend to develop an improved control of the process with the assistance of a computational simulator based on a robust mathematical model of solid-liquid separation. In this article the first stage of this effort is presented: we propose an appropriate framework to model the complex phenomenology observed during sedimentation of flocculated suspensions, and then we assess the behavior and performance of accompanying computational simulations through ample experimental work.

Preliminary industrial-scale studies have reported in general good agreement with the available data. In association with a mining company we aim to validate our results and expand this research towards the industrial consolidation of these tools.

MATHEMATICAL MODEL

We consider an axisymmetric vessel in which the suspension is allowed to settle due to the effect of gravity (see Figure 1a). During the sedimentation process it is possible to distinguish three separate regions within the vessel containing the pulp, namely

- A: clear water zone, containing very low solid content.
- B: hindered settling zone, where particles settle individually and the momentum transfer is performed through the water.
- C: consolidation zone, where particles touch each other forming a network capable of transmitting compression forces.

The phenomenological theory of sedimentation-consolidation, developed in (Bürger, Karlsen & Towers, 2005) and references therein, considers the solid and the fluid as continuous media and assumes that the volume fraction of solid $0 \leq u(x, t) \leq 1$ is solely a function of the thickener height $x$ and time $t$. In particular, $u$ is assumed to be constant across each horizontal cross-section of the vessel. The sedimentation is characterized by the Kynch flux density function $b(u)$, the volume average velocity $Q(t)$ and the solid effective stress $\sigma_s(u)$. The governing equation modeling the solid-liquid separation within a vessel of cross-sectional area $S(x)$ can be written as

$$ (S(x)u)_x + (Q(t)u + S(x)b(u))_x = (S(x)A(u))_x, $$

where $A(u) = \int_0^u a(s)ds$ and $a(u) = (b(u)\sigma_s'(u))/(\Delta g\mu)$. In the last expression $\Delta q$ represents the solid-water density difference and $g$ is the acceleration of gravity.
The model assumes the existence of a parameter $u_{cr}$, called the \textit{critical concentration} or \textit{get point}, which determines the height of the suspension-sediment (B-C) interface. Typically, one considers the constitutive equations $b(u) = v_{w}u(1-u)^{C}$ for $0 \leq u \leq 1$, $\sigma_{e}(u) = 0$ for $0 \leq u \leq u_{c}$ and $\sigma_{e}(u) = \sigma_{0}((u/u_{c})^{k} - 1)$ for $u_{c} < u \leq 1$ (see Figure 1b). In this model $u_{c}, v_{w}, C, \sigma_{0}$ and $k$ are constant material-dependent parameters.

Note that (1) is a \textit{strongly degenerate parabolic differential equation}, meaning that for values of $u < u_{c}$ the equation becomes \textit{hyperbolic}. The time-dependent location of the interface $u = u_{c}$ (the bed height) is unknown beforehand and it is a feature of the solution to the equation.

![Figure 1](image)

(a) Sedimentation zones (b) Flux and stress functions

**Batch mode of sedimentation**

This mode of solid-liquid separation takes place when a closed-bottom vessel is filled with a homogenized suspension. When sedimentation starts, all particles settle at the same velocity forming a water-suspension (A-B) interface that moves downwards (see Figure 2a). Subsequently, a region of sedimentation is formed at the bottom and the suspension-sediment (B-C) interface rises at a certain velocity. As particles continue piling up, we say that the sediment is under compression or consolidation. At a given time, both interfaces meet and the process reaches the equilibrium characterized by an upper region of clear water followed by a downwards increasing concentration gradient.

In the sedimentation-consolidation model, this scenario is represented by the initial condition $u(x, 0) = u_{o}$ for all $0 \leq x \leq L$ (homogeneous suspension) and $Q_{L} = Q_{F} = Q_{R} = 0$, where $Q_{L}, Q_{F}$ and $Q_{R}$ denote the volume overflow, feed and discharge rates, respectively.

**Continuous mode of sedimentation**

Continuous sedimentation if performed in a vessel with a feedwell located at a certain height and overflow/discharge mechanisms at the top/bottom of the container (see Figure 2b). A volume feed rate $Q_{F}(t) \geq 0$ with concentration $u_{F}(t)$ enters through the feedwell and a volume discharge rate $Q_{D}(t) \geq 0$ leaves the tank at the bottom. The underflow concentration $u_{D}(t)$ is a priori unknown.
and it is part of the solution. At the top of the tank clear water leaves at volume flowrate \( Q_o(t) \leq 0 \) with concentration \( u_o(t) \).

If flowrates and concentrations are time-independent, it is possible to reach a steady state. The model includes two different steady states of operation: conventional operation, when the sediment level is located below the feedwell; and high-rate or high-capacity mode, when the sediment level rises into the clarification zone. For simplicity, the model does not consider the effect of the rake located at the bottom of the thickener.

![Figure 2](image_url) (a) Batch mode (b) Continuous mode

**NUMERICAL SIMULATOR**

The numerical scheme introduced to solve (1) is based on the finite difference method to approximate the partial derivatives. The iterative formula can be written explicitly or using a semi-implicit variant for large-time computations. The Engquist-Osher flux is used for the convective part of the equation, ensuring that the finite difference approximation is biased towards the direction from which the information “comes in”. See (Bürger, Karlsen & Towers, 2005). In addition, the appropriate discretization of the problem ensures the numerical stability of the method and convergence to the physically-relevant solution.

The simulator can solve numerically for both batch and continuous modes of operation in transient and steady states. The inputs are the initial concentration \( u_0 \), the critical concentration \( u_c \), the volume flow rates \( Q_F(t), Q_o(t) \) and the parameters associated to the functions of flux density \( b(u) \) and effective solid stress \( \sigma_e(u) \). These parameters are assumed to have been determined by experimental procedures. The container dimensions are introduced through the cross-sectional area function \( S(x) \) allowing for any axisymmetric geometry, such as cylindrical test tubes, clarifier-thickeners, deep cone paste thickeners, etc.

The simulator was entirely implemented in the programming language Python 2.7.10 using the standard libraries for mathematical/statistical tools and output visualization. No additional or commercial software is needed to run the simulations. The user has access to the full data for...
further analysis and use of post-processing tools. After the iterative calculation is finished, several automatically generated figures show different aspects of the solution, including a settling plot (with iso-concentration lines), time-evolution of concentration profiles, 3D plots, among others results.

EXPERIMENTAL VALIDATION

To test the overall performance of the sedimentation simulator we employed laboratory data from a set of sedimentation analyses. The samples were obtained at feed level from industrial tailings thickeners from a Chilean copper ores mine. Table 1 shows the physical properties of the samples. The settling characterization of the pulp was performed through a series of batch tests. The suspension was introduced in a graduate 1 l cylinder, followed by a fixed flocculant dose of 15 g/t. After careful mixing, the height of the clear water-suspension interface was recorded at constant time intervals during the sedimentation. This process was repeated five times (in duplicate) for five different solid concentrations ranging from 9% w/w to 18% w/w. This enables us to estimate five different settling velocities and adjust the data points to the Richardson-Zaki model \( v(u) \) (Richardson & Zaki, 1954), obtaining the parameters \( v_\infty, C \). This also determines the flux density function as \( b(u) = v_\infty u(1 - u)^C \). The critical concentration \( u_c \) and the stress function parameters were estimated using the procedure in (Concha, 2014) and references therein. The values of these constants are shown in Figure 1b.

**Table 1** Physical characterization of the samples (extract)

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>10</td>
<td>-</td>
</tr>
<tr>
<td>Solid density</td>
<td>2730</td>
<td>kg/m³</td>
</tr>
<tr>
<td>D10, D50, D80</td>
<td>4.5, 37.8, 149.9</td>
<td>µm</td>
</tr>
</tbody>
</table>

Figure 3 compares the simulation output (grey scale represents solid concentration) with experimental data (red circles). The numerical solution reproduces the general behavior of the batch test. The calculated clear water-suspension interface is in good agreement with lab data.
Figure 3 Batch test 12% w/w. Comparison between simulation output (grey scale) and lab data (red circles).

The continuous mode of sedimentation of the same pulp was also studied in a semi-pilot settling test. The experimental setup consists of a 1 m tall graduate settling column attached to feed and discharge mechanisms that regulate volume rates. There is also a feedwell cylinder designed with a twofold purpose: facilitate proper flocculant dilution and reduction of feed turbulence. A moving rake at the bottom of the tank transports the material slowly towards the center.

This test was planned to evaluate the performance of an industrial conventional thickener, which was originally designed to reach a discharge concentration of 52% w/w with unitary area equal to 8.64 m²/kg (0.1 tpd). Feed and discharge flows are stepwise adjusted over time to slowly increase the bed height. This process is controlled manually until a steady state is reached. Relevant parameters of the test can be found in Table 2. We obtained a sediment height of 40 cm after 1.5 h (Figure 4a). Measured discharge concentration oscillates around 49% w/w, slightly below the target value of 52% w/w (Figure 4b). The total duration of the test is three hours.

Table 2 Parameters of continuous sedimentation test (extract)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed solid concentration</td>
<td>12</td>
<td>%w/w</td>
</tr>
<tr>
<td>Volume feed rate</td>
<td>1.57E-5</td>
<td>m³/s</td>
</tr>
<tr>
<td>Volume discharge rate</td>
<td>2.64E-6</td>
<td>m³/s</td>
</tr>
</tbody>
</table>
The simulator was used to replicate this experiment using the physical properties of the pulp derived from the laboratory characterization described above. The geometry of the settling column was introduced, along with the target discharge concentration. The simulation output is depicted in Figure 4c. For this example, the simulator was used to determine the bed height that will be needed to achieve the desired discharge concentration. In this case, the suspension-sediment interface must be located slightly above the 55 cm mark for the test to reach a steady state of 52% w/w underflow solid concentration.

The simulation output generally agrees with the settling column data. It also shows that the experiment feed/discharge flow rates should have been slightly increased in order to obtain the necessary bed height. This would have successfully reproduced the thickener’s design performance after one hour of experimental work.

![Figure 4 Continuous mode of sedimentation in settling column. (a) Measured interfaces height (b) Measured discharge concentration (c) Simulation output - Settling plot](image-url)
SCALING-UP

Finally, we tested the performance of the settling simulator using the design parameters of an existing copper tailings clarifier-thickener (see Table 3) and the pulp properties of Table 1. The process was simulated starting from the initial tank fill-up until the desired discharge concentration was attained, approximately 8 h later. The steady state was then maintained for a period of 24 h. The simulation output with the dimensions of the thickener and the obtained bed height are displayed in Figure 5.

Table 3 Clarifier-thickener parameters (extract)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production (design)</td>
<td>0.66 (22 000)</td>
<td>m$^3$/s (tpd)</td>
</tr>
<tr>
<td>Feed solid concentration</td>
<td>31</td>
<td>%w/w</td>
</tr>
<tr>
<td>Discharge solid concentration (design)</td>
<td>56</td>
<td>%w/w</td>
</tr>
<tr>
<td>Volume feed rate</td>
<td>0.57 (19 000)</td>
<td>m$^3$/s (tpd)</td>
</tr>
<tr>
<td>Volume overflow rate</td>
<td>0.340</td>
<td>m$^3$/s</td>
</tr>
</tbody>
</table>

Figure 5 Simulation output – Clarifier-thickener

CONCLUSION

The integration of laboratory data, theoretical models and computational simulations led us to identify a proper framework that reproduces quite well the behavior of batch and continuous sedimentation tests at both laboratory and semi-pilot scale. We have developed and implemented
tools that provide a novel perspective on sedimentation of suspensions, focused on applications to experimental techniques and scalability to the industrial process.

Preliminary applications to full-scale clarifier-thickeners have reported promising results; the next step of this work will be to examine the performance of the model and the computational simulator with sufficient operational data obtained from an industrial plant. We will consider the incorporation of additional variables into the system—such as torque and yield stress—which have shown to play an important role during the thickening process.

The advances of the current model towards industrial validation will involve on-site pulp characterization over diverse periods of operation to take into account the variability that mining slurries frequently undergo. The extension of this research will contribute to the analysis of operational conditions based on a validated mathematical model, reducing uncertainty and improving overall efficiency.

ACKNOWLEDGEMENTS

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NOMENCLATURE

- $b(u)$: Kynch flux density function
- $Q_F, Q_D, Q_O$: volume feed, discharge and overflow rates
- $S(x)$: vessel cross-sectional area
- $\sigma_e(u)$: solid effective stress function
- $u(x, t)$: volume solid fraction
- $u_c$: critical concentration
- $u_F, u_D, u_O$: feed, discharge and overflow concentrations
- %w/w: mass fraction (100% × mass of solute/mass of solution)

REFERENCES

